IMPROVING THE INTERVAL ESTIMATION OF THE QUARTERLY NATIONAL ACCOUNTS OF THE PHILIPPINES

by

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ABSTRACT

This paper aims to suggest alternative methods of interval estimation for the Quarterly National Accounts. At present, Philippine Statistics Authority publishes interval estimates of the GDP growth rate alongside the point estimate of the GDP growth. The confidence interval is computed using the methodology proposed by the paper of Virola and Parcon (1996)[3]. The estimation procedure presented in the said paper constructs a confidence interval using a parameter β, which is a ratio between the preliminary and the revised estimates of the GDP growth. A confidence band is then constructed, creating an upper and lower limits to the GDP growth rate. This methodology is not without issues. First, it assumes a normal distribution for the β, whose probability distribution is unknown. Also, the use of the standard error of the sample mean as a proxy for the true standard error of β could lead to the overstatement or understatement of the standard error used in the interval estimation.

This paper explores a methodology of interval estimation using the bootstrap procedure to identify the distribution of the parameter to be used for the approximation of the confidence interval. This study would also explore a methodology of constructing a confidence interval for GDP levels rather than growth rates.

Keyword: Confidence Interval, GDP, National Accounts

1. The Problem and its Background

The Gross Domestic Product (GDP) is probably the most closely monitored economic indicator. It is often considered as the prime measure of economic performance. Economists, policymakers, businessmen, and the academe are mostly interested in the growth of GDP as it indicates how much the economy has improved from one period to another. The Philippine Statistics Authority (PSA) releases the estimate of the GDP growth 55 days (before 2016, the time lag was 60 days) after the reference quarter. The first release of the GDP growth is preliminary and is subject to multiple rounds of revision. The preliminary GDP growth rate is first revised the following quarter. The revised estimate would again be revised every May, when the statistical agency revises the three-year series of the national accounts, taking into account the availability of new data. In effect, the preliminary estimate would undergo four rounds of revisions: once in the quarter following the release of the preliminary estimate and three times during the succeeding May estimation rounds. For the purposes of this study, we would consider that GDP growth that would no longer be subject to revision as of May 2016 as *final*[4].

Considering the certainty that preliminary estimate of GDP growth would change as new data becomes available, it is imperative for the statistical agency to release interval estimates of the GDP growth. The interval should indicate the range that would contain that final estimate of the GDP growth rates.

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[3] Virola, Romulo A. & Parcon, Gaye A. (1996). On Approximate Confidence Intervals for GDP/GNP Growth Rates. 1996 National Convention on Statistics.

[4] While we refer to GDP growth published as of the writing of this paper "final", note it is still possible for these figures to be revised as a result of conceptual revision and a change in base year, or either.

This paper aims to suggest alternative methods of interval estimation for the Quarterly National Accounts. At present, the PSA publishes interval estimates of the GDP growth rate alongside the point estimate of the GDP growth. The confidence interval is computed using the methodology proposed by the paper of Virola and Parcon (1996). No formal study has been conducted evaluating the performance of this interval. This paper explores a methodology of interval estimation using multiple approaches including the historical approach, the classical least squares and the bootstrap procedures.

The first part of this paper analyzes the revisions of the National Accounts, particularly, the revisions of GDP levels and growth rates at constant prices. The goal of this study is to suggest a methodology for the construction of a prediction interval that would contain the final GDP growth estimate, thus it would be necessary to have a statistical examination of the revisions. This part of the paper attempts to find out if there is a tendency for the GDP to be revised at a certain direction and what is the general magnitude of the revisions. This paper did not, however, cover an analysis of the revisions on the expenditure side of the accounts (Gross Domestic Expenditure), revisions on the gross value added of each industry that makes up the production side, revisions on GDP at current prices, revision on per capita GDP, and revisions on the Gross National Income—all of which deserves a more substantial discussion, perhaps for another study.

The performance of the current methodology for interval estimation of GDP growth rates was also tested. This study examines the final estimate of GDP growth rates falls within the interval released with the preliminary national accounts estimates.

2. Revisions in the national accounts

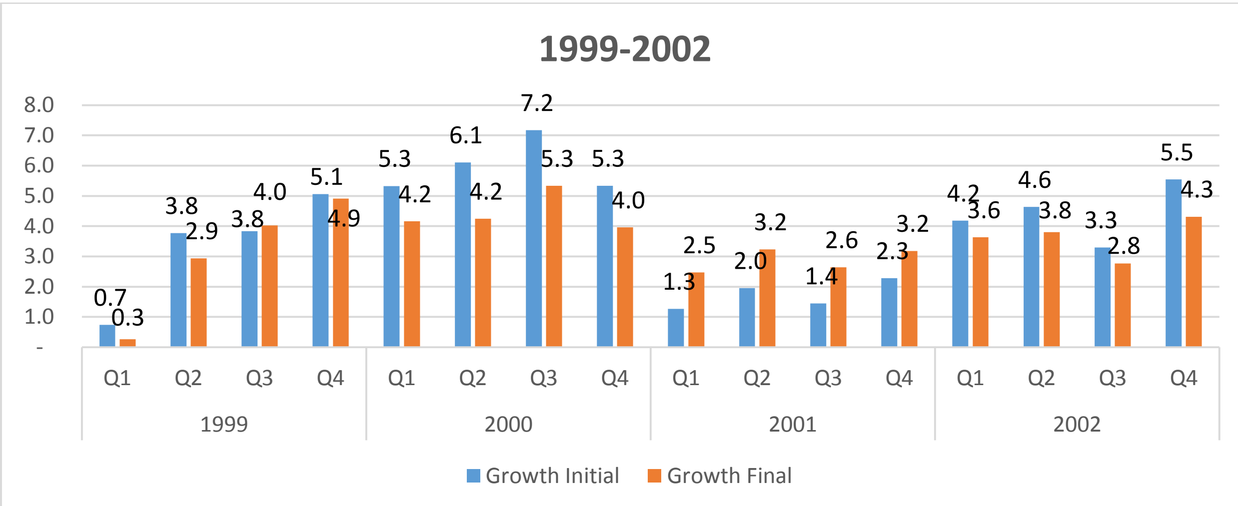
In this section, we analyzed the revisions of the National Accounts of the Philippines from 1999 to 2013. Carson at. Al. (2004) noted four reasons for the revision of official statistics. These are: 1) to incorporate better source data; 2) to capture routine recalculation; 3) to reflect improved methodology, and; 4) to correct errors. Rinne, H. (1969) classified revisions in two categories: statistical and conceptual. Statistical revisions results from the availability of new information. The time lag in the production of data required for the estimation of the National Accounts is usually the reason for statistical revisions. In lieu of the actual and final data, National Accounts statisticians and economists often use preliminary data and apply statistical procedures, and econometric methods to come up with estimates that should be as close as possible to the actual or final value. The availability of census data can also be a source of statistical revisions. As mentioned before, the preliminary estimate of the GDP growth typically undergoes four rounds of revisions. These revisions fall under the category of statistical revisions.

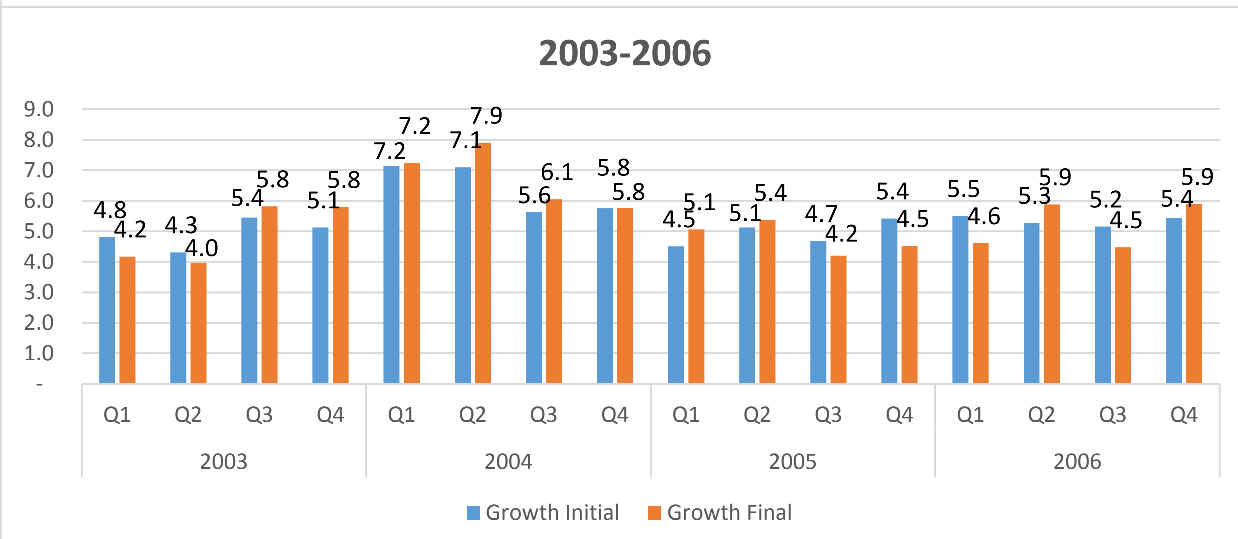
Table 1: Summary statistics of GDP at constant prices from 1999 to 2013

	2000-base GDP Growth at Constant Prices (in percent)					
	1999-2013		**1999-2010**		**2011-2013**	
	Final	**Initial**	**Final**	**Initial**	**Final**	**Initial**
Max	8.9	8.3	8.9	8.3	7.9	7.7
Min	0.3	0.2	0.3	0.2	3.1	3.1
Mean	4.8	4.9	4.6	4.7	5.8	5.8
StDev	2.0	2.0	2.0	2.1	1.7	1.7

The second type of revision has less to do with the data but has more to do with the concepts and definitions used in the compilation of the National Accounts. The System of National Accounts, the manual published by the United Nations for the compilation of National Accounts, had six versions over the years. The earliest publication was in 1953 and the latest version is the 2008 version.

The National Statistical Coordination Board (now the PSA) rebased the GDP in 2011, adopting 2000 as the base year from 1985. As the NSCB revised the time series, the agency also adopted most of the concepts of the 2008 SNA. Thus, revisions prior to 2011 includes revisions due to a change in base year and the adoption of concepts from the 2008 SNA. This makes it difficult to analyze revisions. For the existing series from 1999 to 2010, the statistical and conceptual revisions cannot be distinguished. For these years, we can consider all the revisions to be purely statistical in character.





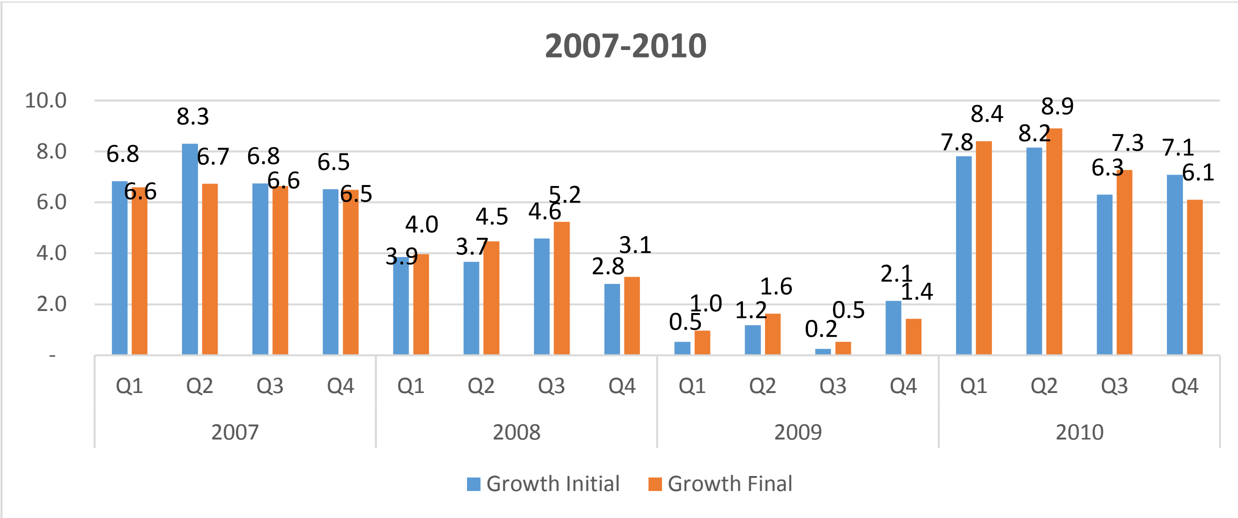


Fig 1.1: Initial GDP growth versus final from 1999 to 2010

From 1999 to 2013, the highest preliminary estimate for the GDP growth rate was for second quarter of 2007, when the initial growth was estimated to be at 8.3 percent. This was revised downward to 6.7 percent. The lowest growth rate recoded from 1999 to 2013 was the 0.2 percent growth in the first quarter of 2009, following the 2008 US stock market crash. This was revised upward to 0.5 percent. The highest growth rate considered final as of May 2016 was for the second quarter of 2016, when growth is recorded to be at 8.9 percent. This was a result of an upward revision from the initial estimate of 8.2 percent. Meanwhile, the lowest growth rate considered final as of May 2016 was for the first quarter of 1999, when the growth is recorded to be 0.3 percent. This was a result of downward revision from the initial estimate of 0.7 percent.

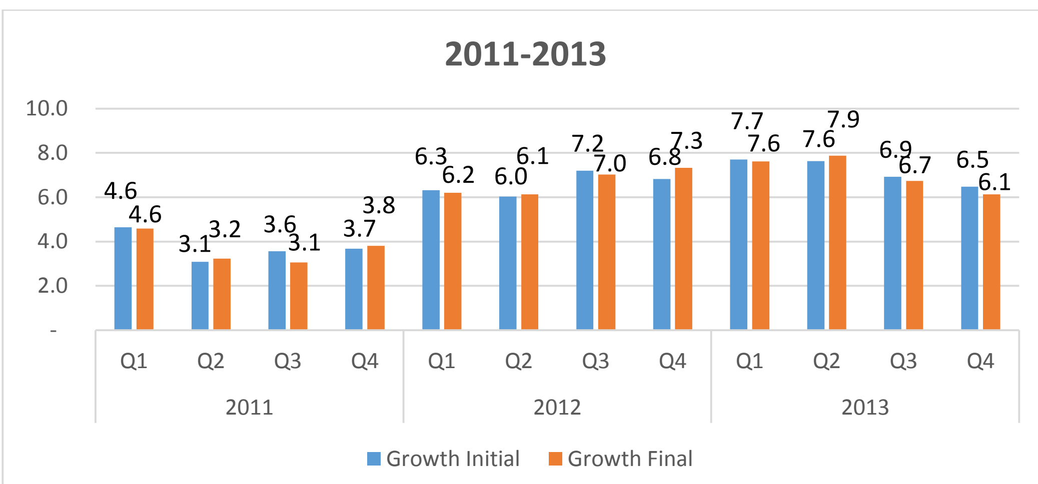


Fig 1.2: Initial GDP growth versus Final from 2011 to 2013

Considering only revisions from preliminary release to the final figure, GDP growth was revised 30 times upward and 30 time downward in the period 1999 to 2013. The highest downward revision was by 1.9 percentage points, which was for the second quarter of 2000, when the GDP growth was revised to 4.2 percent from 6.1 percent. The highest upward revision was by 1.3 percentage point, which was for the second quarter of 2001 when the GDP growth was revised to 3.2 percent from 2.0 percent. The average magnitude of revisions in the GDP grown from 1999 to 2013 was 0.3 percentage points for both upward and downward revisions. Revisions were slightly lower for the years 2011 to 2013. The average revisions for the time period was 0.1 percentage points for both upward and downward revisions. The highest revision from 2011 to 2013 was 0.5 percentage points, which was in the third quarter of 2011 for the series upward revisions and the fourth quarter of 2012 for the series of downward revisions. It could be assumed that since revisions from 2011 to 2013 were purely statistical—a result of updates on the data rather than changes in concepts—it could be expected that revisions would be lower for those years.

Table 2: Summary statistics of GDP growth revisions from 1999 to 2013

	Percentage Points Revision of GDP Growth at Constant Prices					
	1999-2013		1999-2010		2011-2013	
	Downward	Upward	Downward	Upward	Downward	Upward
Max	1.9	1.3	1.9	1.3	0.5	0.5
Mean	0.3	0.3	0.4	0.3	0.1	0.1
StDev	0.5	0.4	0.5	0.4	0.2	0.2
Count	30.0	30.0	23.0	25.0	7.0	5.0

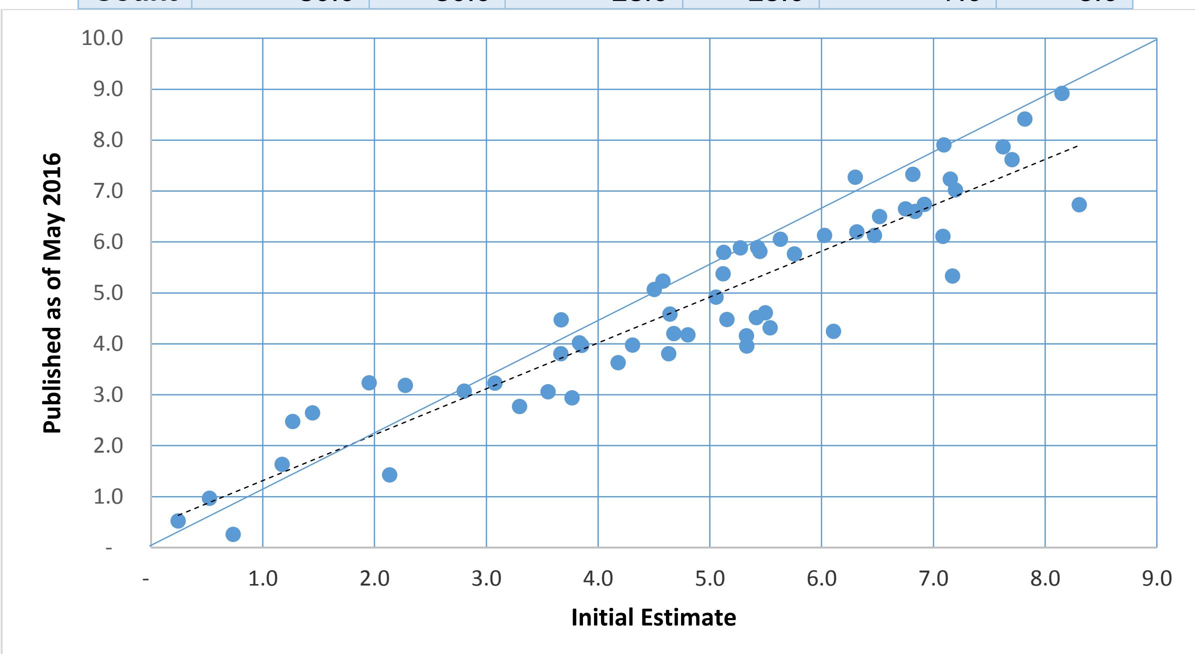


Fig 2: Scatter diagram of GDP growth rates between the initial estimate against the revised

Figure 2 shows a scatter diagram of the initial GDP growth estimate against the final GDP growth estimate as of May 2016. If there were no revisions in the GDP growth, then all points in the scatter plot would fall along the diagonal axis (the blue line). The broken line is the regression line where all points in the graph are centered. It can be noticed that the regression line is lower than the diagonal axis, suggesting a tendency for downward revisions. The broken line can be expressed mathematically as:

$$\widehat{GDP_f} = 0.06 + \ 0.97 * GDP_i \qquad (1)$$

where $\widehat{GDP_f}$ is the estimated final GDP growth rate and GDP_i is the initial GDP growth rate. In this model, the coefficient 0.97 is the factor multiplied to the initial GDP estimate to get an estimate of final GDP growth. It can be noticed that the coefficient is less than 1, suggesting a tendency for downward revisions, given the data from 1999 to 2013. This suggests that the final GDP estimate tends to be 3 percent lower than the initial estimate.

The root mean squared error (RMSE) of the initial GDP growth estimate against the final estimate for 1999 to 2013 is 0.74, the Mean absolute percentage error (MAPE) is 19 percent. Consistent with the authors' expectations, the difference between the final and initial growth rate estimates is larger in the years between 2002 to 2010, when the RMSE is computed to be 0.83 and the MAPE is at 23 percent. On the other hand, RMSE computed for the years between 2011 to 2013 is at 0.26 while the MAPE is computed to be 4 percent. We used these results when we compute for the prediction and confidence intervals.

3. Interval estimation of the National Accounts

The objective of this paper is to propose a methodology for interval estimation of GDP growth rates. The estimation of the Quarterly National Accounts requires data from many sources, making it difficult to derive its variance analytically. There are two considerations in interval estimation: 1) the interval must contain the "true" parameter or prediction that needs to be estimated, and 2) the interval has to be narrow enough to be useful.

Virola & Parcon (2006) proposed the methodology currently being utilized by the PSA for its interval estimation of GDP growth rates. Their methodology uses the average ratio of the initial and final GDP growth rates as the scaling factor for the initial GDP growth. A confidence interval is then created for the scaling factor, assuming it has a normal distribution. The interval is computed using this expression:

$$\{\widehat{\beta_U} = \bar{\beta} + z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\ ;\ \widehat{\beta_L} = \bar{\beta} - z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\} \qquad (2)$$

where

$$\bar{\beta} = \frac{\sum \beta_t}{n} \quad , \qquad (3)$$

$$s = \sqrt{\frac{\sum(\beta_t - \bar{\beta})^2}{n}} \quad , \qquad (4)$$

$$\beta_t = \frac{GDP_f}{GDP_i} \quad , \qquad (5)$$

GDP_f is the final estimate of the GDP growth rate while GDP_i is the initial GDP growth rate estimate. The upper and lower limit of the scaling factor β is then multiplied to the initial GDP growth to arrive at the upper and lower limit of the GDP growth rates as shown in equation 5:

$$\{\widehat{GDP_U} = \widehat{\beta_U} * GDP_i\ ;\ \widehat{GDP_L} = \widehat{\beta_L} * GDP_i\ \} \qquad (6)$$

The interval being generated by the PSA is essentially a function of the revisions. If the average magnitude of the revisions is large, the resulting scaling factor β would be large as well. Extensive variations in the revisions also affect the interval. Large variances in the gap between the final and initial GDP estimates would result to large standard errors of the scaling factor, which would ultimately cause the spread of the interval to be wide. It is also worth noting that the data used in the generation of the interval does not discriminate between statistical and conceptual revisions. This implies that the interval generated for GDP growth also attempts to capture the final GDP growth resulting from conceptual revisions.

In this paper, two general approaches were considered. First, we utilize a constant Root Mean Squared Error to generate prediction intervals for the GDP growth. This approach is noted in literature as the Historical Approach. The second approach employed the ordinary least squares procedure in the generation of intervals for the GDP growth. Time series data on the initial and final GDP growth estimates from 1999 to 2011 was used in the generation of the interval. The same parameters was be used to create intervals from 2012 to 2013. For both strategies, the bootstrap procedure was be employed as an addition avenue for analysis. The bootstrap procedure usually generates superior results when the distribution of the variable of interest is unknown.

Similar to the methodology being employed by the PSA, the data used in the exercise did not discriminate between statistical and conceptual revisions. The GDP growth published as of the writing of this paper was considered final.

A. Historical Method

The most common approach in constructing intervals when using time series data is the historical method (Stoke et. Al.s, 2014). This procedure involves maintaining a constant measure of bias in the computation of the interval. It assumes that the errors are normally distributed with the mean of zero and the standard deviation equal to the RMSE. The upper and the lower limit of the GDP growth is computed as follows:

$$\{\widehat{GDP_{Ut}} = GDP_{it} + z_{\frac{\alpha}{2}} * RMSE(k);\ \widehat{GDP_{Lt}} = GDP_{it} - z_{\frac{\alpha}{2}} * RMSE(k)\} \quad (7)$$

where:

$$RMSE(k) = \sqrt{\frac{\sum(GDP_{it} - GDP_{ft})^2}{n}} \quad (8)$$

and GDP_{it} is the initial estimate of the GDP growth at time t, GDP_{ft} is the final estimate of the GDP growth at time t and n is the number of observation or the length of the time series.

B. Ordinary Least Squares Estimation

This method follows the same general principle as the methodology currently being employed by the PSA. However, instead of using the average ratio between the initial and final GDP growth as a scaling factor for the initial GDP growth estimate, regression parameters were used. The ordinary least squares method was utilized having the final GDP growth as the endogenous variable and the initial GDP growth estimate would be exogenous.

$$GDP_{ft} = \beta_0 + \beta_1 GDP_{it} + \hat{u}_t \quad (9)$$

where GDP_{ft} is the final estimate at time t of the GDP growth rate, GDP_{it} is the initial GDP growth rate estimate and $\hat{u}_t$ is the stochastic error terms which is assumed to be normally distributed with a constant variance. The parameters for the regression were the scaling factor and a confidence interval was generated for the parameters of the regression:

$$\{\widehat{\beta_{iU}} = \beta_i + z_{\frac{\alpha}{2}} * SE;\ \widehat{\beta_{iL}} = \beta_i - z_{\frac{\alpha}{2}} * SE\} \quad (10)$$

where SE is the standard error of the coefficients. The predictions intervals were based on the upper and lower limits of the regression parameters. The intervals were calculated using the expression below:

$$\{\widehat{GDP_{Ut}} = \widehat{\beta_{0U}} + \widehat{\beta_{1U}} * GDP_{it};\ \widehat{GDP_{Lt}} = \widehat{\beta_{0L}} + \widehat{\beta_{1L}} * GDP_{it}\} \quad (11)$$

C. Bootstrap Procedure

The bootstrap procedure, in principle, approximates the sampling distribution by repeatedly drawing samples from the population of interest. We used the bootstrap procedure to estimate intervals for the regression parameters. A similar approach was utilized by Simionescu (2014) in generating prediction intervals for inflation and unemployment in Romania.

Two bootstrap methods were used in the regression models: the bootstrap residuals approach and bootstrap by pairs approach. The first approach starts from the bootstrap model:

$$Y^* = X\hat{\beta} + u^* \quad (12)$$

where u* is a stochastic term taken from residuals û of the original regression. The selected sample is $\{y_i^*\}$ where i is from 1 to n. The stochastic term of the bootstrap process makes use of the modified residuals:

$$\tilde{u}_i = \frac{\hat{u}_i}{\sqrt{1-h_i}} - \frac{1}{n}\sum_s^n \frac{\hat{u}_s}{\sqrt{1-h_s}} \quad (13)$$

The theoretical bootstrap process can be expressed as:

$$y_t(b) = X_t\hat{\beta} + \tilde{u}_t(b) \quad (14)$$

where t = 1, 2…, n, b is the order of iteration and $\tilde{u}_t(b)$ is resampled from $\hat{u}_t$ (Simionescu, 2014; Juan & Lantz, 2001). In this approach, the explanatory variables in matrix X were considered fixed. This approach, however, do not produce optimal results in the presence of heteroskdasticity (Juan & Lantz, 2001).

The bootstrap by pairs approach involves the direct resampling of the data set used in the original data. The danger with the use of this procedure is that it may cause endogeneity, the occurrence of correlations between regressors and the error terms (Juan & Lantz, 2001). GRETL 2016a was used in the calculations.

4. Empirical Results

As indicated in the description of the methodologies, the intervals generated are functions of the revisions in the GDP growth rates. If the revisions are large, it follows that the spread of the intervals would also likely be large.

a. PSA Methodology

An evaluation of the interval being published by the PSA shows that the 95 percent interval failed to capture the final GDP growth 29 times from 2002 to 2013. In particular, the final GDP growth went beyond the lower limit of the GDP growth eight times from 2002 to 2013. The upper limit of the interval, meanwhile, failed to capture the final GDP growth 21 times from 2002 to 2013. The average range of the interval was 0.7 percentage points, which implies that on average, revisions exceed 0.35 percentage points would not be captured by the interval.

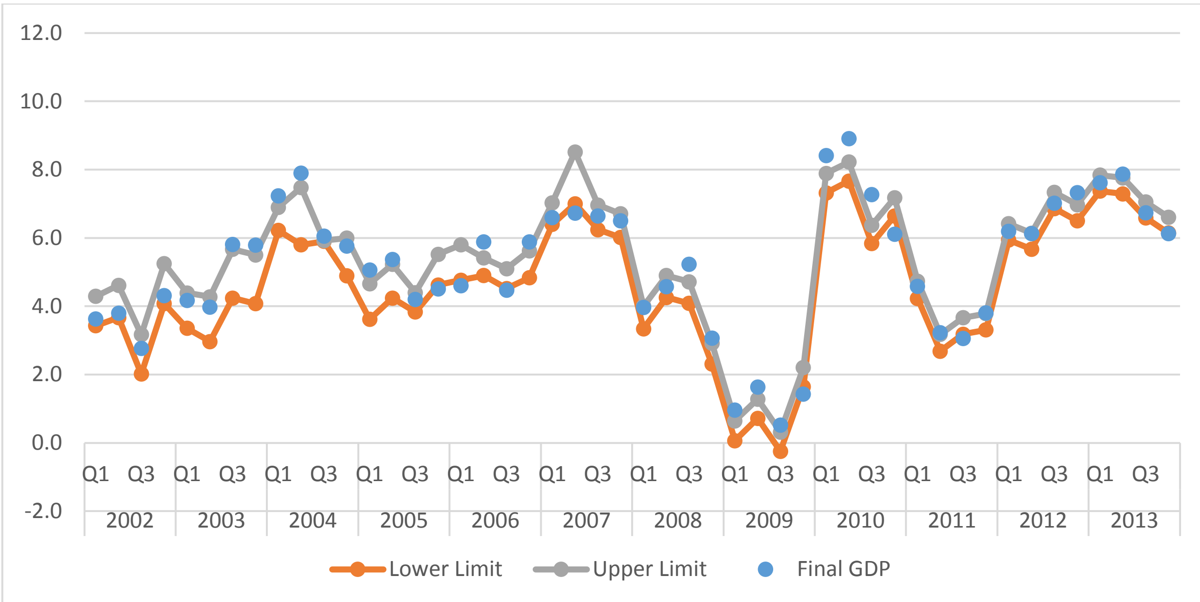


Fig 3: Plot of the Final GDP Estimate and the 95% Confidence Interval Produced by the PSA

For the period 2012 to 2013, the period characterized only by statistical revisions, the interval generated by the methodology of the PSA failed to capture the final GDP growth six times. In particular, the final GDP growth went beyond the lower limit twice. Meanwhile, the final GDP growth went beyond the upper limit four times.

b. Historical Approach

In this method, the RMSE was used to estimate the GDP growth rate prediction intervals. The RMSE was computed using equation 6

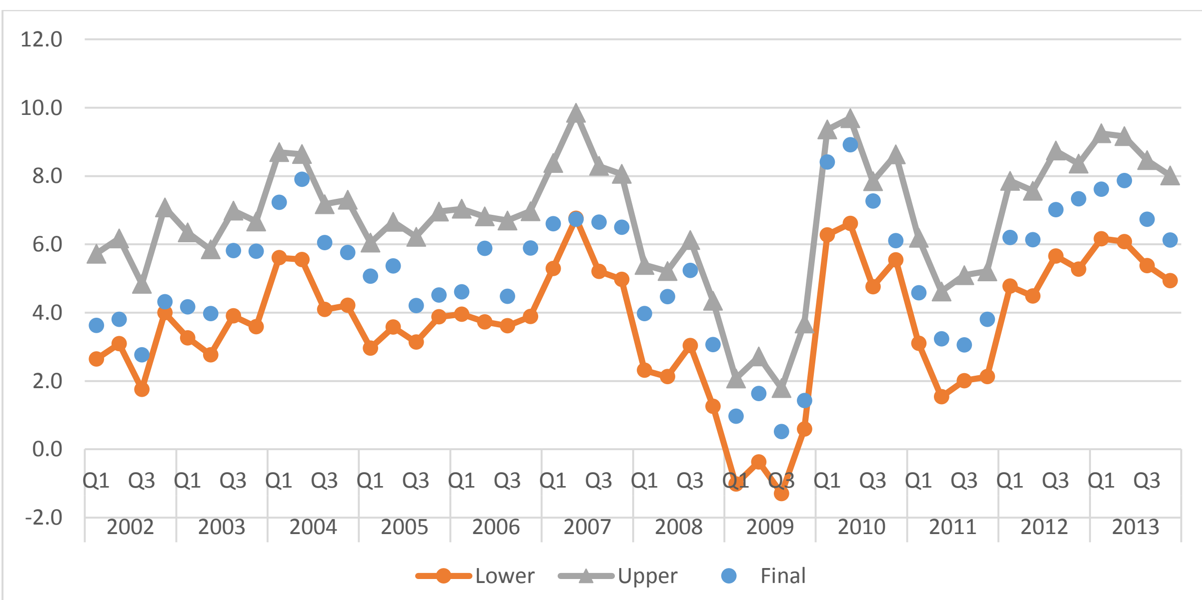


Fig 4.1: Plot of the Final GDP Estimate and the 95% Confidence Interval using the Historical Method

Figure 4.1 and 4.2 show the intervals using a constant RMSE. From 2002 to 2013, the interval created using the historical approach failed to capture the final GDP growth only once, which was in the second quarter of 2007, when the GDP growth was revised downward from 8.3 percent to 6.7 percent or 1.6 percentage points. For the period 2012 to 2013, the period wherein revisions are only statistical in nature, the intervals generated for both techniques were able to capture the final GDP growth 100 percent of the time.

c. Ordinary Least Squares Approach

Ordinary least squares was applied to the data from 1999 to 2011 using the model in equation 8. The resulting regression equation is a follows:

$GDP_f = 0.06 + 0.97 * GDP_i$ (15)

Fig 5.1: Plot of the Final GDP Estimate and the 95% Confidence Interval using OLS

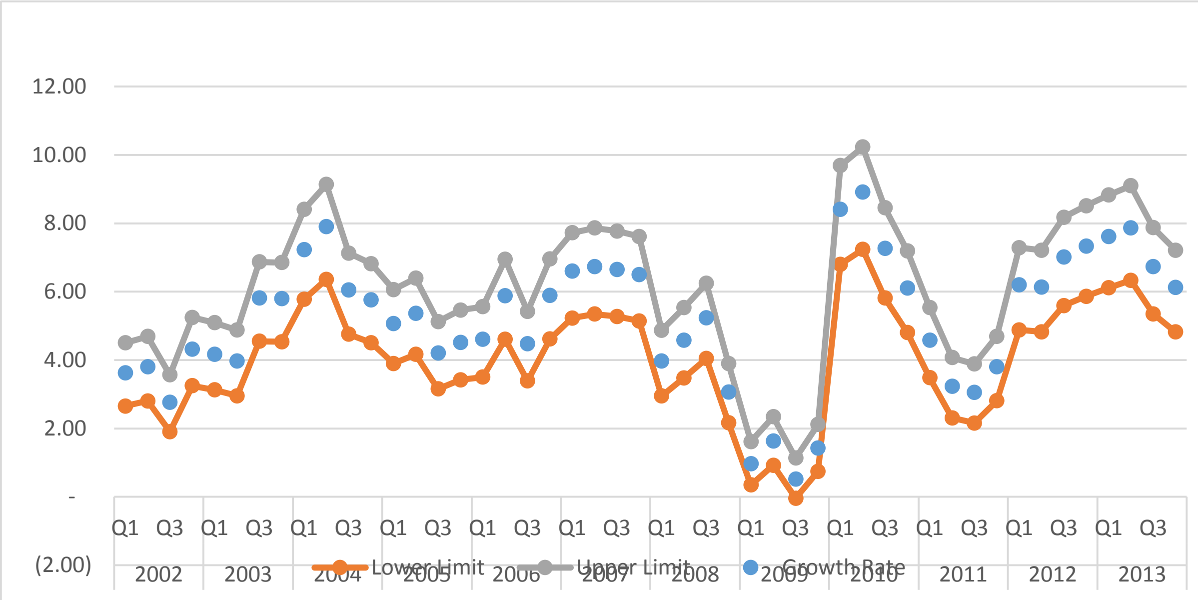


Fig 5.2: Plot of the Final GDP Estimate and the 95% Confidence Interval using Bootstrap Residuals

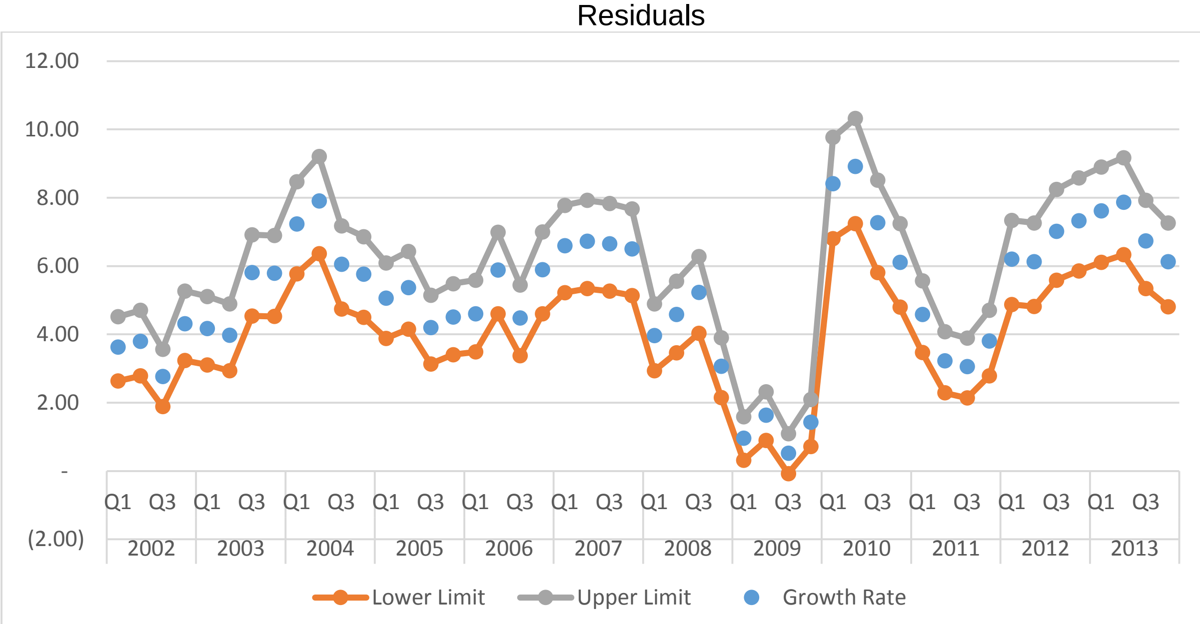


Fig 5.3: Plot of the Final GDP Estimate and the 95% Confidence Interval using XY Pairs

It can be noticed that the resulting regression equation is identical to the results in equation 1. This suggests that the functional relationship between the initial and the final GDP is the same from the period 1999 to 2011 and the longer period which is from 2002 to 2013. The 95 percent intervals generated from this approach captured the final GDP 100 percent of the time. The approach yielded an average spread of 1.6 percentage points.

As mentioned in Part III.b, intervals were also generated using the bootstrap residual and the bootstrap pairs approach. In both techniques, the intervals were able to capture the final GDP growth 100 percent of the time. The average spread of both intervals is 2.2 percentage points.

d. Comparing the Results

A comparison between the four approaches and the methodology currently being utilized by the PSA can be seen in Table 3. In terms of capturing the final GDP growth estimates, the intervals generated by the four approaches performed better compared to the methodology currently being employed by the PSA. As mentioned before, the methodology being employed by the PSA was not able to capture the final GDP growth 29 times from 2002 to 2013. The historical approach was not able to capture the final GDP growth only once while the intervals using the OLS-based approaches were able to capture the final GDP growth 100 percent of the time.

Table 3: Comparison of Spread and Ability of the Intervals to Capture the Final GDP Growth

95 percent Confidence Interval					
	Current Method	**Historical Approach**	**OLS**	**Bootstrap Residuals**	**Bootstrap XY Pairs**
Number of Misses	29	1	-	-	-
Average Spread	0.7	3.1	1.6	2.2	2.2

Comparing the average spread of the intervals, the methodology being employed by the PSA possessed the narrowest spread, at 0.7 percentage points. The methodologies based on the historical approach yielded widest spread, at 3.1 percentage points. Of the three approaches which captured the final GDP growth 100 percent of the time from 2002 to 2013, the technique using the classical OLS has the narrowest average spread, at 1.6 percentage points. The bootstrap residual and bootstrap XY pairs approaches both generated intervals with a spread of 2.2 percentage points.

5. Conclusions and Recommendations

Considering the uncertainty of knowing the final GDP given only the preliminary estimate, it is imperative for the statistics agency compiling the National Accounts to release intervals that would more or less indicate the "true" performance of the economy. An evaluation of the interval estimates of the PSA reveals the short comings of the methodology the agency currently employs. Out of the four methodologies attempted in this exercise, the intervals generated from the OLS (without bootstrap) yielded the most favorable results. It was able to capture the final GDP growth, 100 percent of the time while having a spread that is narrower compared to the intervals generated by the other methodologies.

The spread of 1.6 percentage points is still large, considering that most forecast intervals of GDP growth has a spread of only 1.0 percentage points. The spread of the interval generated by the PSA with only 0.7 percentage points, substantially narrower compared to the intervals generated from the OLS approach. One of the reason why the spread of the intervals tends to be large is the magnitude of revisions. For all methodologies, the intervals are only a function of the revisions. If the revisions are large, the intervals to be generated would likely be wide. The wide intervals could be indicative that revisions are large. As mentioned in Part II of this paper, revisions could be as large as 1.9 percentage points. Despite having a spread that is relative wide, the authors believe that the interval generated by the OLS approach is the most appropriate.

It is worth noting that the data used in this study does not discriminate between statistical and conceptual. In effect, it is implied that the intervals generated by the methodologies employed in this study also attempts to capture the final GDP resulting from both conceptual and statistical revisions.

This raises the question: should interval estimates of the GDP growth attempt to capture the final GDP growth resulting from conceptual revisions or should it only capture the final GDP growth resulting from statistical revisions? Future studies on this topic could explore this problem.

Given the results, we believe that it is critical to have a more thorough study on the revisions of the national accounts in order to generate better intervals for the GDP growth. The magnitude of the revisions could differ for every sector of the GDP. It may be necessary to generate different intervals for each of these sectors. Future research in the field could also consider the Bayesian approach to interval estimation, as it tends to generate superior results from limited time series data.

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Appendix 1: Interval Estimates of GDP Growth Rates using PSA methodology

		GDP	Confidence Intervals for the True Growth Rate of GDP					
Period		Growth[1/]	90%		95%		99%	
		Final GDP	Lower Limit	Upper Limit	Lower Limit	Upper Limit	Lower Limit	Upper Limit
2002	Q1	3.6	3.5	4.2	3.4	4.3	3.3	4.4
	Q2	3.8	3.7	4.5	3.7	4.6	3.5	4.8
	Q3	2.8	2.1	3.1	2.0	3.2	1.8	3.3
	Q4	4.3	4.2	5.2	4.1	5.2	3.9	5.4
2003	Q1	4.2	3.4	4.3	3.4	4.4	3.2	4.6
	Q2	4.0	3.1	4.2	3.0	4.3	2.8	4.5
	Q3	5.8	4.4	5.6	4.2	5.7	4.0	5.9
	Q4	5.8	4.2	5.4	4.1	5.5	3.9	5.7
2004	Q1	7.2	6.1	7.5	6.2	6.9	6.0	7.8
	Q2	7.9	6.3	7.4	5.8	7.5	6.0	7.7
	Q3	6.1	4.8	6.6	5.9	5.9	4.6	6.1
	Q4	5.8	5.0	5.9	4.9	6.0	4.8	6.2
2005	Q1	5.1	3.7	4.6	3.6	4.7	3.5	4.8
	Q2	5.4	4.3	5.1	4.2	5.2	4.2	5.4
	Q3	4.2	3.6	4.7	3.8	4.4	3.7	4.5
	Q4	4.5	5.5	5.5	4.6	5.5	4.5	5.7
2006	Q1	4.6	5.0	5.6	4.8	5.8	4.8	5.9
	Q2	5.9	4.6	5.3	4.9	5.4	4.8	5.5
	Q3	4.5	4.3	5.3	4.5	5.1	4.4	5.2
	Q4	5.9	4.3	5.0	4.8	5.6	4.1	5.2
2007	Q1	6.6	6.3	7.2	6.4	7.0	6.3	7.2
	Q2	6.7	7.8	7.8	7.0	8.5	6.9	8.0
	Q3	6.6	6.3	6.9	6.2	7.0	6.1	7.1
	Q4	6.5	6.1	6.7	6.0	6.7	5.9	6.8
2008	Q1	4.0	3.4	3.9	3.3	4.0	3.2	4.1
	Q2	4.6	4.3	4.8	4.3	4.9	4.2	5.0
	Q3	5.2	4.1	4.7	4.1	4.7	4.0	4.8
	Q4	3.1	2.4	2.9	2.3	2.9	2.2	3.0
2009	Q1	1.0	0.1	0.6	0.1	0.6	0.0	0.7
	Q2	1.6	0.8	1.2	0.7	1.3	0.6	1.4
	Q3	0.5	-0.2	0.3	-0.2	0.3	-0.3	0.4
	Q4	1.4	1.7	2.2	1.6	2.2	1.6	2.3

2010	Q1	8.4	7.4	7.8	7.3	7.9	7.2	8.0
	Q2	8.9	7.7	8.2	7.7	8.2	7.6	8.3
	Q3	7.3	5.9	6.3	5.8	6.4	5.8	6.5
	Q4	6.1	6.7	7.1	6.6	7.2	6.6	7.3
2011	Q1	4.6	4.3	4.7	4.2	4.7	4.1	4.8
	Q2	3.2	2.7	3.1	2.7	3.2	2.6	3.3
	Q3	3.1	3.2	3.6	3.2	3.7	3.1	3.7
	Q4	3.8	3.3	3.7	3.3	3.8	3.2	3.9
2012	Q1	6.2	6.0	6.4	5.9	6.4	5.9	6.5
	Q2	6.1	5.7	6.1	5.7	6.1	5.6	6.2
	Q3	7.0	6.9	7.3	6.9	7.3	6.8	7.4
	Q4	7.3	6.5	6.9	6.5	7.0	6.4	7.0
2013	Q1	7.6	7.4	7.8	7.4	7.8	7.3	7.9
	Q2	7.9	7.3	7.7	7.3	7.8	7.2	7.8
	Q3	6.7	6.6	7.0	6.6	7.1	6.5	7.1
	Q4	6.1	6.2	6.6	6.1	6.6	6.1	6.7

Appendix 2: Interval Estimates of GDP Growth Rates using Historical Approach

Quarter	Growth Rate		Square Error	Prediction Interval					
				90%		95%		99%	
	Initial	Final		Lower	Upper	Lower	Upper	Lower	Upper
Q1:2002	4.2	3.6	0.3	2.9	5.5	2.6	5.7	2.2	6.2
Q2:2002	4.6	3.8	0.7	3.3	5.9	3.1	6.2	2.6	6.7
Q3:2002	3.3	2.8	0.3	2.0	4.6	1.8	4.8	1.3	5.3
Q4:2002	5.5	4.3	1.5	4.2	6.8	4.0	7.1	3.5	7.6
Q1:2003	4.8	4.2	0.4	3.5	6.1	3.3	6.3	2.8	6.8
Q2:2003	4.3	4.0	0.1	3.0	5.6	2.8	5.9	2.3	6.3
Q3:2003	5.4	5.8	0.1	4.2	6.7	3.9	7.0	3.4	7.5
Q4:2003	5.1	5.8	0.4	3.8	6.4	3.6	6.7	3.1	7.2
Q1:2004	7.2	7.2	0.0	5.9	8.4	5.6	8.7	5.1	9.2
Q2:2004	7.1	7.9	0.6	5.8	8.4	5.6	8.6	5.1	9.1
Q3:2004	5.6	6.1	0.2	4.3	6.9	4.1	7.2	3.6	7.7
Q4:2004	5.8	5.8	0.0	4.5	7.1	4.2	7.3	3.7	7.8
Q1:2005	4.5	5.1	0.3	3.2	5.8	3.0	6.0	2.5	6.5
Q2:2005	5.1	5.4	0.1	3.8	6.4	3.6	6.7	3.1	7.1
Q3:2005	4.7	4.2	0.2	3.4	6.0	3.1	6.2	2.7	6.7
Q4:2005	5.4	4.5	0.8	4.1	6.7	3.9	7.0	3.4	7.4
Q1:2006	5.5	4.6	0.8	4.2	6.8	4.0	7.0	3.5	7.5
Q2:2006	5.3	5.9	0.4	4.0	6.6	3.7	6.8	3.2	7.3
Q3:2006	5.2	4.5	0.5	3.9	6.4	3.6	6.7	3.1	7.2
Q4:2006	5.4	5.9	0.2	4.1	6.7	3.9	7.0	3.4	7.5
Q1:2007	6.8	6.6	0.1	5.5	8.1	5.3	8.4	4.8	8.9
Q2:2007	8.3	6.7	2.5	7.0	9.6	6.8	9.8	6.3	10.3
Q3:2007	6.8	6.6	0.0	5.5	8.0	5.2	8.3	4.7	8.8
Q4:2007	6.5	6.5	0.0	5.2	7.8	5.0	8.1	4.5	8.5
Q1:2008	3.9	4.0	0.0	2.6	5.1	2.3	5.4	1.8	5.9
Q2:2008	3.7	4.5	0.6	2.4	5.0	2.1	5.2	1.6	5.7
Q3:2008	4.6	5.2	0.4	3.3	5.9	3.0	6.1	2.6	6.6
Q4:2008	2.8	3.1	0.1	1.5	4.1	1.3	4.3	0.8	4.8
Q1:2009	0.5	1.0	0.2	-0.8	1.8	-1.0	2.1	-1.5	2.6
Q2:2009	1.2	1.6	0.2	-0.1	2.5	-0.4	2.7	-0.9	3.2
Q3:2009	0.2	0.5	0.1	-1.0	1.5	-1.3	1.8	-1.8	2.3
Q4:2009	2.1	1.4	0.5	0.8	3.4	0.6	3.7	0.1	4.2
Q1:2010	7.8	8.4	0.3	6.5	9.1	6.3	9.4	5.8	9.8
Q2:2010	8.2	8.9	0.6	6.9	9.4	6.6	9.7	6.1	10.2
Q3:2010	6.3	7.3	0.9	5.0	7.6	4.8	7.8	4.3	8.3
Q4:2010	7.1	6.1	1.0	5.8	8.4	5.5	8.6	5.1	9.1
Q1:2011	4.6	4.6	0.0	3.3	5.9	3.1	6.2	2.6	6.7
Q2:2011	3.1	3.2	0.0	1.8	4.4	1.5	4.6	1.1	5.1

Q3:2011	3.6	3.1	0.2	2.3	4.8	2.0	5.1	1.5	5.6
Q4:2011	3.7	3.8	0.0	2.4	5.0	2.1	5.2	1.6	5.7
Q1:2012	6.3	6.2	0.0	5.0	7.6	4.8	7.9	4.3	8.3
Q2:2012	6.0	6.1	0.0	4.7	7.3	4.5	7.6	4.0	8.1
Q3:2012	7.2	7.0	0.0	5.9	8.5	5.7	8.7	5.2	9.2
Q4:2012	6.8	7.3	0.3	5.5	8.1	5.3	8.4	4.8	8.8
Q1:2013	7.7	7.6	0.0	6.4	9.0	6.2	9.2	5.7	9.7
Q2:2013	7.6	7.9	0.1	6.3	8.9	6.1	9.2	5.6	9.7
Q3:2013	6.9	6.7	0.0	5.6	8.2	5.4	8.5	4.9	8.9
Q4:2013	6.5	6.1	0.1	5.2	7.8	4.9	8.0	4.4	8.5

Appendix 3.1: Summary of Regression Results for the OLS Approach

Dependent Variable: FINAL
Method: Least Squares
Sample: 2002Q1 2011Q4
Included observations: 40

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.064873	0.279596	0.232023	0.8178
INITIAL	0.975527	0.053172	18.34665	0

R-squared	0.898558	Mean dependent var	4.861794
Adjusted R-squared	0.895889	S.D. dependent var	1.941612
S.E. of regression	0.626486	Akaike info criterion	1.951325
Sum squared resid	14.9144	Schwarz criterion	2.035769
Log likelihood	-37.0265	Hannan-Quinn criter.	1.981857
F-statistic	336.5994	Durbin-Watson stat	1.488418
Prob(F-statistic)	0		

Appendix 3.2: Confidence Interval of Regression Parameters for OLS Approach

Coefficient Confidence Intervals
Sample: 2002Q1 2011Q4
Included observations: 40

		90% CI		**95% CI**		**99% CI**	
Variable	**Coefficient**	**Low**	**High**	**Low**	**High**	**Low**	**High**
C	0.06	-0.41	0.54	-0.50	0.63	-0.69	0.82
INITIAL	0.98	0.89	1.07	0.87	1.08	0.83	1.12

Appendix 4: Interval Estimates of GDP Growth Rates using OLS Approach

Quarter	Growth Rate		Prediction Interval					
			90%		95%		99%	
	Initial	Final	Lower	Upper	Lower	Upper	Lower	Upper
Q1:2002	4.2	3.6	3.4	4.9	3.3	5.0	3.0	5.3
Q2:2002	4.6	3.8	3.8	5.4	3.7	5.5	3.4	5.8
Q3:2002	3.3	2.8	2.6	3.9	2.5	4.1	2.2	4.4
Q4:2002	5.5	4.3	4.7	6.3	4.5	6.5	4.1	6.8
Q1:2003	4.8	4.2	4.0	5.5	3.8	5.7	3.5	6.0
Q2:2003	4.3	4.0	3.5	5.0	3.4	5.2	3.1	5.5
Q3:2003	5.4	5.8	4.6	6.2	4.4	6.4	4.1	6.7
Q4:2003	5.1	5.8	4.3	5.9	4.1	6.0	3.8	6.4
Q1:2004	7.2	7.2	6.1	8.0	5.9	8.2	5.5	8.6
Q2:2004	7.1	7.9	6.1	8.0	5.9	8.2	5.5	8.5
Q3:2004	5.6	6.1	4.7	6.4	4.6	6.6	4.2	6.9
Q4:2004	5.8	5.8	4.8	6.6	4.7	6.7	4.3	7.1
Q1:2005	4.5	5.1	3.7	5.2	3.6	5.4	3.3	5.7
Q2:2005	5.1	5.4	4.3	5.9	4.1	6.0	3.8	6.4
Q3:2005	4.7	4.2	3.9	5.4	3.7	5.6	3.4	5.9
Q4:2005	5.4	4.5	4.5	6.2	4.4	6.4	4.0	6.7
Q1:2006	5.5	4.6	4.6	6.3	4.4	6.4	4.1	6.8
Q2:2006	5.3	5.9	4.4	6.0	4.2	6.2	3.9	6.5
Q3:2006	5.2	4.5	4.3	5.9	4.1	6.1	3.8	6.4
Q4:2006	5.4	5.9	4.6	6.2	4.4	6.4	4.1	6.7
Q1:2007	6.8	6.6	5.8	7.7	5.6	7.9	5.3	8.3
Q2:2007	8.3	6.7	7.2	9.3	7.0	9.5	6.5	9.9
Q3:2007	6.8	6.6	5.8	7.6	5.6	7.8	5.2	8.2
Q4:2007	6.5	6.5	5.5	7.4	5.4	7.5	5.0	7.9
Q1:2008	3.9	4.0	3.1	4.5	3.0	4.7	2.7	5.0
Q2:2008	3.7	4.5	3.8	5.3	3.6	5.5	3.3	5.8
Q3:2008	4.6	5.2	3.8	5.3	3.6	5.5	3.3	5.8
Q4:2008	2.8	3.1	2.2	3.4	2.0	3.6	1.8	3.8
Q1:2009	0.5	1.0	0.1	1.0	0.0	1.1	-0.2	1.3
Q2:2009	1.2	1.6	0.7	1.7	0.6	1.8	0.4	2.0
Q3:2009	0.2	0.5	-0.2	0.7	-0.2	0.8	-0.4	1.0
Q4:2009	2.1	1.4	1.6	2.7	1.4	2.8	1.2	3.1
Q1:2010	7.8	8.4	6.7	8.7	6.5	8.9	6.1	9.3
Q2:2010	8.2	8.9	7.0	9.1	6.8	9.3	6.4	9.7
Q3:2010	6.3	7.3	5.3	7.1	5.2	7.3	4.8	7.7
Q4:2010	7.1	6.1	6.1	8.0	5.9	8.2	5.5	8.5
Q1:2011	4.6	4.6	3.8	5.4	3.7	5.5	3.4	5.8
Q2:2011	3.1	3.2	2.4	3.7	2.3	3.8	2.0	4.1

Q3:2011	3.6	3.1	2.8	4.2	2.7	4.4	2.4	4.6
Q4:2011	3.7	3.8	3.0	4.3	2.8	4.5	2.5	4.8
Q1:2012	6.3	6.2	5.4	7.1	5.2	7.3	4.8	7.7
Q2:2012	6.0	6.1	5.1	6.8	4.9	7.0	4.6	7.4
Q3:2012	7.2	7.0	6.2	8.1	6.0	8.3	5.6	8.7
Q4:2012	6.8	7.3	5.8	7.7	5.6	7.9	5.3	8.2
Q1:2013	7.7	7.6	6.6	8.6	6.4	8.8	6.0	9.2
Q2:2013	7.6	7.9	6.5	8.5	6.3	8.7	5.9	9.1
Q3:2013	6.9	6.7	5.9	7.8	5.7	8.0	5.3	8.4
Q4:2013	6.5	6.1	5.5	7.3	5.3	7.5	5.0	7.9

Appendix 5: Confidence Interval of Regression Parameters for Bootstrap Approach

For the coefficient on **Initial** (point estimate 0.975434):	Lower	Upper
95% confidence interval = 0.867095 to 1.08381	0.867095	1.08381
Based on 1999 replications, using **resampled residuals**		
For the coefficient on **const** (point estimate 0.0647953):	Lower	Upper
95% confidence interval = -0.49089 to 0.571498	-0.49089	0.571498
Based on 1999 replications, using **resampled residuals**		
For the coefficient on **Initial** (point estimate 0.975434):	Lower	Upper
95% confidence interval = 0.871372 to 1.09894	0.871372	1.09894
Based on 1999 replications, using **resampled y,X "pairs"**		
For the coefficient on **const** (point estimate 0.0647953):	Lower	Upper
95% confidence interval = -0.526812 to 0.52626	- 0.526812	0.52626
Based on 1999 replications, using **resampled y,X "pairs"**		

Appendix 6: Interval Estimates of GDP Growth Rates using Bootstrap Procudures

Quarter	Growth Rate		Residuals 95%		XY Pairs 95%	
	Initial	Final	Lower	Upper	Lower	Upper
Q1:2002	4.2	3.6	2.7	4.5	2.6	4.5
Q2:2002	4.6	3.8	2.8	4.7	2.8	4.7
Q3:2002	3.3	2.8	1.9	3.6	1.9	3.6
Q4:2002	5.5	4.3	3.2	5.2	3.2	5.3
Q1:2003	4.8	4.2	3.1	5.1	3.1	5.1
Q2:2003	4.3	4.0	3.0	4.9	2.9	4.9
Q3:2003	5.4	5.8	4.5	6.9	4.5	6.9
Q4:2003	5.1	5.8	4.5	6.9	4.5	6.9
Q1:2004	7.2	7.2	5.8	8.4	5.8	8.5
Q2:2004	7.1	7.9	6.4	9.1	6.4	9.2
Q3:2004	5.6	6.1	4.8	7.1	4.7	7.2
Q4:2004	5.8	5.8	4.5	6.8	4.5	6.9
Q1:2005	4.5	5.1	3.9	6.1	3.9	6.1
Q2:2005	5.1	5.4	4.2	6.4	4.2	6.4
Q3:2005	4.7	4.2	3.2	5.1	3.1	5.1
Q4:2005	5.4	4.5	3.4	5.5	3.4	5.5
Q1:2006	5.5	4.6	3.5	5.6	3.5	5.6
Q2:2006	5.3	5.9	4.6	6.9	4.6	7.0
Q3:2006	5.2	4.5	3.4	5.4	3.4	5.4
Q4:2006	5.4	5.9	4.6	7.0	4.6	7.0
Q1:2007	6.8	6.6	5.2	7.7	5.2	7.8
Q2:2007	8.3	6.7	5.3	7.9	5.3	7.9
Q3:2007	6.8	6.6	5.3	7.8	5.3	7.8
Q4:2007	6.5	6.5	5.1	7.6	5.1	7.7
Q1:2008	3.9	4.0	3.0	4.9	2.9	4.9
Q2:2008	3.7	4.5	3.5	5.5	3.5	5.6
Q3:2008	4.6	5.2	4.0	6.2	4.0	6.3
Q4:2008	2.8	3.1	2.2	3.9	2.1	3.9
Q1:2009	0.5	1.0	0.3	1.6	0.3	1.6
Q2:2009	1.2	1.6	0.9	2.3	0.9	2.3
Q3:2009	0.2	0.5	0.0	1.1	-0.1	1.1
Q4:2009	2.1	1.4	0.7	2.1	0.7	2.1
Q1:2010	7.8	8.4	6.8	9.7	6.8	9.8
Q2:2010	8.2	8.9	7.2	10.2	7.2	10.3
Q3:2010	6.3	7.3	5.8	8.5	5.8	8.5
Q4:2010	7.1	6.1	4.8	7.2	4.8	7.2
Q1:2011	4.6	4.6	3.5	5.5	3.5	5.6

Q2:2011	3.1	3.2	2.3	4.1	2.3	4.1
Q3:2011	3.6	3.1	2.2	3.9	2.1	3.9
Q4:2011	3.7	3.8	2.8	4.7	2.8	4.7
Q1:2012	6.3	6.2	4.9	7.3	4.9	7.3
Q2:2012	6.0	6.1	4.8	7.2	4.8	7.3
Q3:2012	7.2	7.0	5.6	8.2	5.6	8.2
Q4:2012	6.8	7.3	5.9	8.5	5.9	8.6
Q1:2013	7.7	7.6	6.1	8.8	6.1	8.9
Q2:2013	7.6	7.9	6.3	9.1	6.3	9.2
Q3:2013	6.9	6.7	5.4	7.9	5.3	7.9
Q4:2013	6.5	6.1	4.8	7.2	4.8	7.3